

Name _____ Date _____ Partners _____

ELECTRON CHARGE-TO-MASS RATIO

OBJECTIVES

- To understand how electric and magnetic fields impact an electron beam
- To experimentally determine the electron charge-to-mass ratio

OVERVIEW

In this experiment, you will measure e/m , the ratio of the electron's charge e to its mass m . Given that it is also possible to perform a measurement of e alone (the Millikan Oil Drop Experiment), it is possible to obtain the value of the mass of the electron, a very small quantity.

If a particle carrying an electric charge q moves with a velocity \mathbf{v} in a magnetic field \mathbf{B} that is at a right angle to the direction of motion, it will experience the **Lorentz force**:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (1)$$

Which, because of the vector product, is always perpendicular to both the magnetic field and the direction of motion. A constant force that is always perpendicular to the direction of motion will cause a particle to move in a circle. We will use this fact to determine e/m of the electron by measuring the radius of that circle. To this end we must:

- produce a narrow beam of electrons of known energy,
- produce a uniform magnetic field,
- find a way to measure the radius r of the circular orbit of the electrons in that magnetic field,

and

- find the relation between that radius and the ratio e/m .

We discuss these problems in order.

The Electron Beam

When one heats a piece of metal, say a wire, to 1,000 K or beyond, electrons will "boil off" from its surface. If one surrounds the wire with a positively charged electrode, an **anode**, the electrons will be attracted to it and move radially outward as indicated in Fig. 1. On their way to the anode they will acquire a kinetic energy

$$E_k = \frac{1}{2}mv^2 = eV, \quad (2)$$

where V is the potential difference, or voltage, between the heated filament, called the **cathode**, and the anode. Most of the electrons will strike the anode. However, if one cuts a narrow slit into the anode, those electrons that started out toward the slit will exit through it as a narrow beam with a kinetic energy E_k .

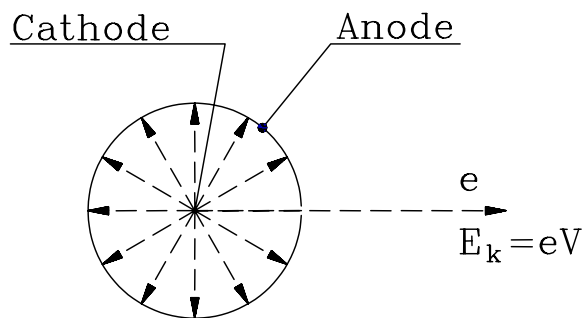


Fig. 1. Electron gun. Electrons are produced by the hot cathode at the center. A voltage V between the cathode and anode accelerates the electron through a hole in the anode, so the electrons have kinetic energy eV .

The Magnetic Field

According to Ampere's law a wire carrying a current I is surrounded by a magnetic field \mathbf{B} , as shown in Fig. 2. If the wire is bent into a circle the field lines from all sides reinforce each other at the center, creating an axial field (see Fig. 3). Usually one will not use a single loop of wire to

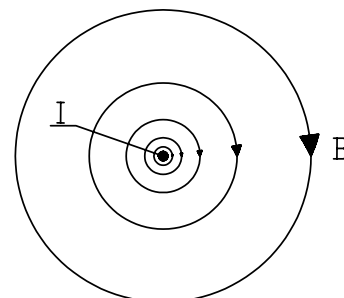
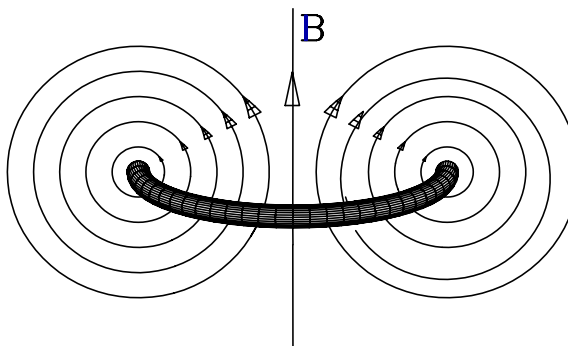


Fig. 2. Magnetic field of a straight wire.

Fig. 3. Magnetic field of a wire loop.



create a field but a coil with many turns.

If one uses two coaxial coils of radius R that are a distance d apart, as shown in Fig. 4, the field at the center point between the coils will be nearly homogeneous.¹ H. von Helmholtz (1821-1894) realized that there remains a free parameter, namely the coil separation d , that can still be adjusted. He showed that when $d = R$, the result is a particularly homogeneous field in the central region between the coils.² Since that time, **Helmholtz coils** have been used when there is a need for homogeneous magnetic fields.

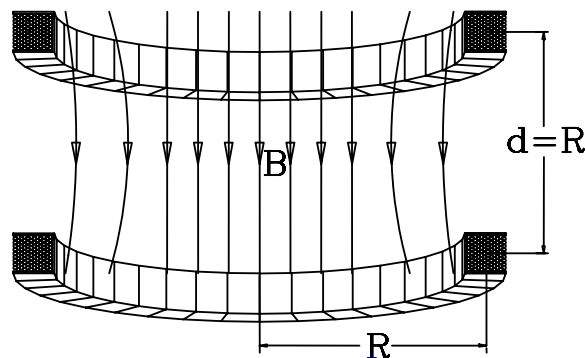


Fig. 4. Magnetic field \mathbf{B} of a pair of Helmholtz coils. R is the radius of the coils and d is the spacing between them.

One can show that the field in the center of a Helmholtz coil is given by

$$B = \frac{8NI \mu_0}{5R\sqrt{5}}, \quad (3)$$

where I is the current flowing through both coils, R is their mean radius, N is the number of turns of wire in each coil, and $\mu_0 = 4\pi \times 10^{-7} \text{ T}\cdot\text{m/A}$ is the **permeability constant**.

The Electron Orbit

Experiments like the one that you will perform have been used to measure the mass of charged particles with great precision. In these experiments the particles move in a circular arc whose beginning and end are measured very accurately in a near perfect vacuum. For our simple experiment we cannot go to such lengths and a simple expedient has been used to make the electron orbit visible. The bulb surrounding the electron source is filled with either mercury (larger apparatus) or helium (smaller apparatus) vapor. When the electrons collide with the atoms, the atoms emit light so that one can follow the path of the electron beam. These collisions will diminish the accuracy of the experiment but it remains adequate for our purposes.

¹ The symmetry of the arrangement makes the first derivative of the field with respect to the axial direction vanish.

² The second derivative vanishes as well.

A particle moving in a circle of radius r must be held there by a **centripetal force**

$$F_c = \frac{mv^2}{r}. \quad (4)$$

In our case, that centripetal force is the Lorentz force, Eq. (1), hence

$$evB = \frac{mv^2}{r} \quad (5)$$

This equation contains the velocity v , which we can eliminate by using Eq. (2). Rewriting Eq. (2), we find

$$v = \sqrt{\frac{2eV}{m}} \quad (6)$$

and hence

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \quad (7)$$

The uncertainty in e/m is determined by the uncertainty in V , B , and r .

$$\left(\frac{\sigma_{e/m}}{e/m}\right)^2 = \left(\frac{\sigma_V}{V}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + \left(\frac{\sigma_r}{r}\right)^2 \quad (8)$$

In Equations (7) and (8), V is the voltage between cathode and anode and r is the mean radius of the circular electron orbit, both of which can be measured, and B is the magnetic field through which the electrons pass. We know the magnetic field at the center of the Helmholtz coil, which can be obtained, using Eq. (3), from a measurement of the current through the coils, the dimension of the coils and the number of turns. The magnetic field does not change very much away from the center of the coils.

INVESTIGATION 1: FINDING e/m

For this investigation, you will need the following:

- Daedalon Helmholtz coil with e/m glass tube
- Bar magnet
- Meter stick

Activity 1-1: The Helmholtz Coil Apparatus

In this activity, you will familiarize yourself with the setup you will be using. See Fig. 5 for a schematic of the apparatus.

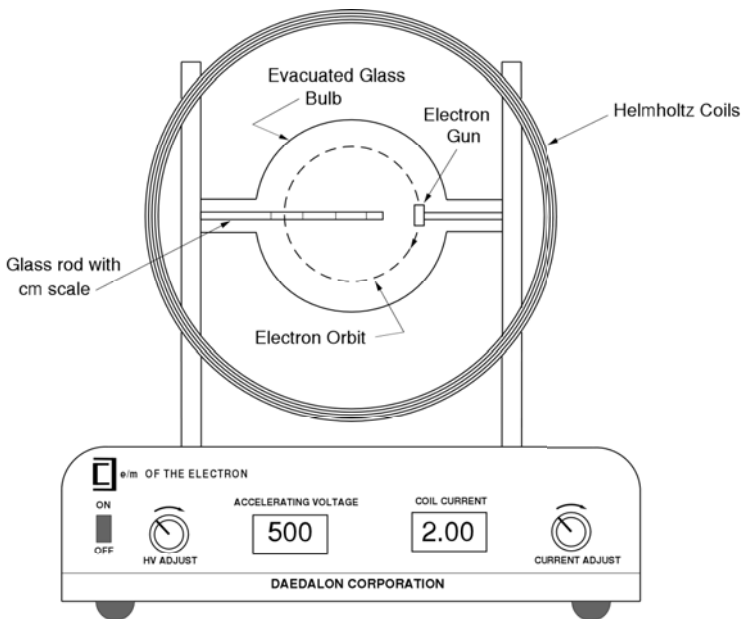


Fig. 5: e/m Apparatus

1. Turn the main power on. The unit will perform a self test lasting no more than 30 s. Do not do anything with the unit during the self test. When it is finished, the coil current display will be stabilized and indicate "000". The unit is now ready to use, but note that **there is a 10 minute warm-up time before you should take final measurements.** You can go ahead and proceed with the remainder of this procedure.
2. Look in the center of the Helmholtz coils for the glass tube. **The tube is extremely fragile, and the wires sticking out from the tube have current and voltage, so be very careful.** This evacuated glass tube has helium vapor inside. The electrons will follow circular orbits inside this tube.
3. Locate the grid and anode inside the glass tube. It is the pair of vertically oriented metal cylinders with a gap between them. At its top and center is the filament or cathode that will be heated by a current to emit the electrons.

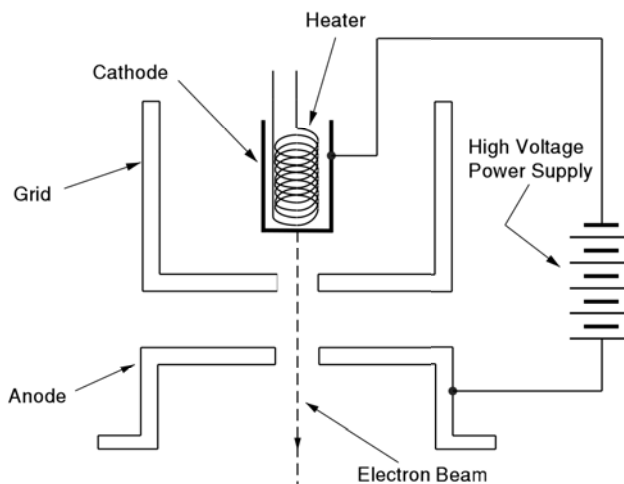


Fig. 6: Grid and Anode

4. There are three separate electrical circuits (see Fig. 6): 1) to heat the filament/cathode (over which you have no control); 2) to apply a voltage between cathode and anode (denoted as Accelerating Voltage on the unit); 3) to supply the current (denoted as Coil Current) for the Helmholtz coils.
5. Measure the diameter of the Helmholtz coils in several places and take the average. Record the mean radius R below.

Radius R _____

6. Measure the mean separation d between the coils. You may want to average several measurements here also.

Coil separation d _____

Verify that $d \approx R$. (Note: We have found they may not quite agree.)

7. The manufacturer states that there are 130 turns in each coil.
8. Calculate the “constant of proportionality” between the current passing through the Helmholtz coils and the magnetic field produced. You will need the above parameters to do this. Look at Eq. (3).

$$B_{\text{Helmoltz}} (\text{Tesla}) = \underline{\hspace{2cm}} \times I (\text{amps})$$

Question 1-1: The nominal value for the apparatus you will be using is 1.3 mT (milliTesla). The Earth's magnetic field is approximately 0.1 mT and is pointing into the ground at an angle of about 58° with respect to the horizontal. Discuss how much difficulty the Earth's magnetic field will cause in your experiment.

Question 1-2: In what direction should the apparatus be aligned to minimize the effects of the Earth's magnetic field? Explain.

NOTE: We cannot compensate for the Earth's magnetic field with this apparatus.

9. Turn up the voltage adjust knob to a voltage of 150 V. Look for the electron beam, which should be pointing down. Turn up the current adjust knob and look at the path of the electron beam.
10. You should now be able to see the filament (two wires that are orange due to being heated).

NOTE: Both the voltage and current outputs are controlled by a microprocessor, which locks out the controls at both the minimum and maximum settings. There is not a manual "stop" on the knobs. When the knob reaches the maximum setting, it will still turn, but the appropriate value will not change. This feature prevents excessive voltage being applied to the tube or current through the coils.

11. Turn the Current Adjust control up and observe the circular deflection of the beam. When the current is high enough, the beam will form a circle. The diameter of the electron path in the magnetic field can be measured using the etched glass internal scale in the tube. The graduations and numerals of the scale are illuminated by the collision of the electrons, making observation reading fairly easy. Vary the Current Adjust and note the electron beam striking several of the centimeter scale markings. You should also be able to see a vertical

line indicating the half-centimeter mark. The scale numbers fluoresce when the beam hits them. Set the beam on the 11 cm mark.

NOTE: Sometimes the electron beam slightly misses the internal glass scale tube. If it is really bad, ask your TA to look at it. **DO NOT TRY TO MAKE ANY ADJUSTMENTS!** Sometimes it helps to put a dim backlight behind the scale to help see the numbers and half-centimeter marks.

Question 1-3: What happens to the beam of electrons as the coil current is increased?

Question 1-4: What causes the phenomena that you just described in Question 1-3?

12. Set the coil current to 1.7 A. Adjust the accelerating voltage while looking at the electron beam path. Do not exceed 350 V.

Question 1-5: What do you observe as the accelerating voltage is changed while keeping the coil current constant? Explain why this occurs.

13. While the electron beam is somewhere near the middle of your vertical targets, use the bar magnet to see how it affects the electron beam.

Question 1-6: Describe what you observe as you move the bar magnet around. Can you produce a helical path for the electron? Can you see why you want to keep spurious magnetic fields away from the electron beam?

14. Make sure you place the bar magnet at the other end of the table from your apparatus to minimize any possible effects.

Activity 1-2: Measurement Of Charge-To-Mass Ratio

NOTE: The electrons collide with the helium atoms introduced to make the beam visible. Unavoidably, the electrons lose some energy in the collisions (that make the beam visible). In order to minimize this effect, you should concentrate on those electrons that have the highest energy: those at the outer edge of the beam (largest radius). Ask your Instructor if you are uncertain about this.

1. We will make measurements for the electrons moving in various circular orbits. You will write down the diameter of the path by reading the illuminated tube. You will need these in order to determine e/m from Eq (7).
2. Set the anode voltage to 150 V. Change the coil current until the beam hits the 5 cm mark. This will be the diameter, not the radius. Record the coil current and diameter in **Table 1-1**. Increase the coil current in turn to go through each 0.5 cm of the diameter. The 0.5 cm marks are a vertical line between the numbered cm marks. This can proceed quite rapidly by one person observing the beam while changing the coil current and another person filling in **Table 1-1**.
3. Proceed in turn to observe and record data for the other accelerating voltages in **Table 1-1**. Sometimes the beam will physically not be present for every position listed in **Table 1.1**. Leave the table blank when this occurs.
4. **Turn the main power off.**

Table 1-1

	Accelerating (anode) Voltage				
	150 V	200 V	250 V	300 V	350 V
Diameter (cm)	Coil Current (A)	Coil Current (A)	Coil Current (A)	Coil Current (A)	Coil Current (A)
5.0					
5.5					
6.0					
6.5					
7.0					
7.5					
8.0					
8.5					
9.0					
9.5					
10.0					

- Set up a table in Excel to calculate the values for e/m for each of your data points. **We suggest columns for electron path diameter and radius, accelerating voltage, coil current, magnetic field, and anything else you think you need to calculate e/m using Eq. (7). Enter the voltages one after the other in the same column.** Don't forget to change your values to the SI system before determining your final value of e/m . The last column should be the difference between your value of e/m and the accepted one (1.759×10^{11} C/kg).
- Average all your results in Excel to obtain a mean value for e/m .
 e/m _____
- Find the statistical error of your average value. (*Hint*: Think "standard error of the mean")
Uncertainty _____
- Compare your value with the accepted value of e/m . Percentage difference: _____%

Question 1-7: How well does your result compare with the accepted value?

9. **Print** out one data table for your group. You only need one printout per group.

Activity 1-3: Investigation of Systematic Error

Question 1-8: What could be the sources of error in your experiment?

It is useful in an experiment like this to see if there are systematic errors that might affect your final results.

1. Look at your answer to Question 1-8. It may be possible that we may learn something about our errors if we compare the values of e/m versus our parameters. Because we have our data in Excel, it is quite easy to do that.
2. Plot all your values of e/m versus a) accelerating voltage, b) coil current, and c) orbit radius. Produce trendline fits for your data.
3. **Print** out one copy each of these graphs and include them in your report.

Question 1-8: Look carefully at the three plots you just made and at your trendlines. Do you see any patterns? Could these possibly indicate any systematic problems? Discuss these possible errors and how you might be able to correct for them or improve the experiment. List any other systematic errors you can think of and discuss.

<p>PLEASE CLEAN UP YOUR LAB AREA AND MAKE SURE THAT THE POWER IS OFF!</p>
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